

B.A./B.Sc. - Ist Sem. (Old Syllabus 2014)

(2118)

Mathematics Paper-II

(Calculus-I)

Time allowed: 3 hrs.

Max. Marks: 50

Note: Attempt five questions in all, selecting atleast TWO questions from each section. All questions carry equal marks.

SECTION-A

1. a) Prove that $\sqrt{7}$ is not a rational number.

b) Solve $\frac{x-2}{x+2} < \frac{x+1}{x-1}$ (5,5)

2.a) Prove that union of two bounded sets is a bounded set. What can you say about its converse?

Justify your answer.

b) Prove that $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist (5,5)

3) a) Prove that $\lim_{x \rightarrow \frac{1}{2}} [1 + x + x^2] = 1$

b) For what values of a and b will the following function be continuous $\forall x$?

$$f(x) = \begin{cases} 13, & x \leq 2 \\ ax^2 + bx + 1, & 2 < x < 3 \\ 17 - ax, & x \geq 3 \end{cases}$$

c) Show that $f(x) = x^2 + 5$ is uniformly continuous on $[0,1]$. (3,4,3)

4. a) Find $\frac{dy}{dx}$, if $y = x^{\cosh x} + (\sinh x)^{2x}$

b) State and prove Squeeze Principle. (5, 5)

5) a) Give an example of a field which is not complete. Justify your answer.

b) Prove that $2 \tanh^{-1} \left(\tan \frac{x}{2} \right)$ and $\cosh^{-1}(\sec x)$ can only differ by a constant. (5, 5)

SECTION-B

6) a) if $p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$ prove that $p + \frac{a^2 p}{a^2} = \frac{a^2 b^2}{p^3}$

b) if $y = \cos(m \sin^{-1} x)$, find $y_n(0)$ (5,5)

7) a) Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}}$

b) Use Maclaurin's Theorem with Lagrange's form of remainder to expand $\sin x$ as far as the n^{th} term in terms of ascending powers of x . (5,5)

8) a) State and prove Leibnitz's Theorem.

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b) Show that the points of inflexion of the curve $y^2=(x-a)^2(x-b)$ lie on the line $3x+a=4b$.

(5,5)

9) a) Find the position and nature of double points of the curve $y^2=(x-1)(x-2)^2$

b) Find the asymptotes of the curve

$$3x^3 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0$$

Show that the asymptotes meet the curve again at three points which lie on a line. Find its equation.

(5,5)

10. a) Trace the curve $y = \frac{x}{1+x^2}$

b) Find the curvature at the point $(\frac{3a}{2}, \frac{3a}{2})$ on the curve $x^3 + y^3 = 3axy$ (5,5)

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